Design of Infinite Impulse Response Digital Differentiators using PSO, BAT, and FPA Optimization Techniques

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**Abstract.** This research gives a thorough investigation into the design and performance evaluation of IIR digital differentiators using three major nature-inspired metaheuristic algorithms: Particle Swarm Optimization (PSO), BAT Algorithm (BAT), and Flower Pollination Algorithm (FPA). The problem is framed as an optimization task in which the filter coefficients are set to reduce the frequency domain error between the ideal and designed differentiators. Extensive simulations are performed to evaluate the frequency response, phase properties, and approximation error of the developed differentiators. The collected findings show that these optimization techniques provide significant gains in accuracy when compared to traditional approaches, with FPA outperforming in terms of convergence speed and error minimization. The work emphasizes the potential of bio-inspired algorithms in enhancing digital filter designs and sheds light on their usefulness in real-time signal processing applications.

# Introduction

Digital differentiators are utilized for computing the time derivative of an input signal and are described as , where denotes the Frequency Range of operation [1],[2],[3][4]. Many digital signal processing applications require digital differentiators, and these devices are also used in a range of other applications. Consequently, there is a great deal of interest among academics to create lower-order digital differentiators that can be used in real-time processing applications [5].

Rabiner and Gold proposed building differentiators that could be either non-recursive or recursive in the early 1970s [3],[4]. The design of IIR-type digital differentiators was fundamentally altered in 1992 by the method proposed by Al-Alaoui [6], [7], [ 8], [9]. An important goal of this study is to develop digital differentiators of the IIR kind.

Infinite Impulse Response (IIR) digital differentiators are critical in signal processing applications such as edge detection, biomedical signal analysis, and control systems. Their design involves optimizing coefficients to achieve desired frequency responses, often framed as non-convex optimization problems. Traditional methods (e.g., least squares, Remez exchange) face challenges in balancing accuracy, stability, and computational efficiency. Metaheuristic algorithms like PSO, BAT, and FPA have emerged as powerful alternatives [10]-[13].

Aggarwal et al. [14] proposed an -norm based design for IIR digital differentiators and integrators using BAT. The method minimized phase response errors and outperformed conventional gradient-based techniques, demonstrating BAT’s efficacy in frequency-domain optimization. Gupta et al. [15] designed wideband digital integrators and differentiators using PSO, optimizing magnitude response errors. Their approach outperformed window-based methods, showcasing PSO’s capability in multi-objective filter design [16]. Combined BAT with constraint handling for stability, demonstrating robustness in noisy environments [17],[18]. Liu et al. [19] introduced a BA with “moderate orientation and perturbation of trend” to prevent premature convergence. This variant enhanced search diversity by dynamically adjusting velocity and position updates. Amalo et al. [20] developed the Cultural Bat Algorithm (CBA), integrating cultural evolution principles to refine solution quality. CBA improved convergence speed and accuracy in benchmark tests [21[,[22],[23]. Zhang et al. [24] first applied PSO to IIR filter design, demonstrating superior convergence over genetic algorithms (GA). Focused on magnitude error minimization.

The document is structured as described below. In the following section (Section 2), the notion of digital differentiators is discussed. An explanation is given for the mathematical derivation that was based on the method that Al-Alaoui suggested. In the third section, a proposal is made for the design of innovative differentiators that are based on the concept of model order reduction. In Section 4, we present the various application areas in which the proposed filters could be used. In the fifth and last section, conclusions are drawn.

# Conventional Methods

The time derivative of the input signal is computed using digital differentiators. Digital differentiators can be classified as either finite or infinite impulse response types. This essay primarily focuses on the construction of digital differentiators utilizing meta-heuristic methodologies. A digital differentiator comprises two categories: finite impulse response (FIR) and infinite impulse response (IIR). Numerous papers have attempted to quantify the design of various filter types along with their respective advantages and disadvantages. Among IIR-type differentiators, Al-Alaoui’s has demonstrated superior efficacy. The inversion and stabilization provided by the digital integrator provide the basis of this endeavor. Some of the conventional digital differentiators and integrators are tabulated in Table 1.

**TABLE 1.** Digital differentiators and integrators

|  |  |
| --- | --- |
| **Name** | **Digital Differentiator** |
| **First Order** |  |
| Backward |  |
|  |  |
| AL Alaoui |  |
| Tahar |  |
| Guran Stannic |  |
| **Second Order** |  |
| Simpsons |  |
| Tick |  |
| AL Alaoui Second Order |  |
| **Third Order** |  |
| Ngo |  |

# Metaheuristic Techniques

Glover [11] first used the term “meta-heuristic” to describe a collection of approaches that govern the construction of heuristics and are hence conceptually superior to heuristics. If an optimization problem can be solved with a sufficiently good lower-level procedure or heuristic, then the higher-level method or heuristic can be called a meta-heuristic. When compared to calculus-based approaches or simple heuristics, meat-heuristics typically find better solutions with less computational cost by searching over a broader set of potential options.

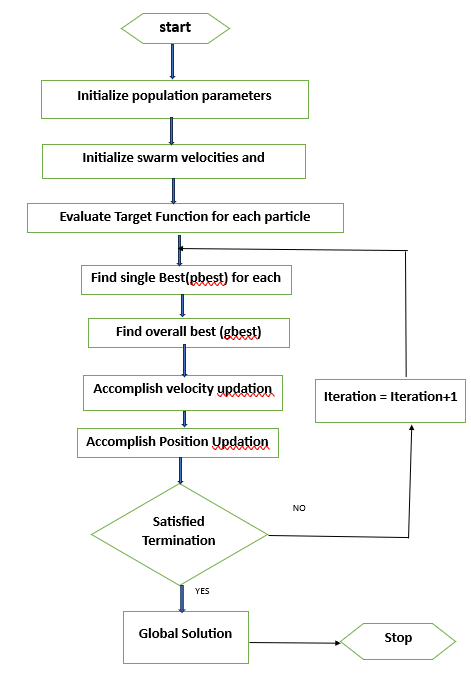
## Particle Swarm Optimization (PSO)

The PSO was conceived in 1995 [10] (see Figure 1). It is an optimization method derived from the behavior of avian or aquatic species. PSO can be delineated into three components: Algorithms, Topology, and Parameters. PSO was originally designed for real-value problems, but it can be adapted to address binary and discrete issues [16]. In mathematical terms, the population of PSO () with N particles evolves step-by-step from the starting point () to a complete generation () of iterations. A vector of d dimensions is represented by each particle in . The target function () is used to determine the grade of every particle. Particle velocities are revised mathematically by means of the aforementioned formula.

The updation of the search is,

*Where*

With each iteration, the cognitive knowledge that each particle has stored about its optimal position (pbest) is updated. Every iteration updates the social information that each particle has stored, gbest .



**FIGURE 1.** Flowchart of PSO

## Flower Pollination Algorithm (FPA)

Yang designed FPA in 2012, which outperformed other metaheuristic algorithm. Before discussing the FPA algorithm, let’s first look at how plants pollinate. Figure 2 illustrates two types of pollination.

1. Pollinators, like as insects and birds, are involved in biotic or cross-pollination. Velocities and speeds of pollinators vary greatly, but they cover great distances. This is a worldwide pollination process with Levy flight characteristics. This pollination method is used by approximately 90% of all blooming plants globally.
2. In abiotic pollination, also known as self-pollination, no outside pollinators are required. This pollination technique is employed by around 10% of plant species. When pollinators, like wind, cover short distances, it results in pollination at the local level.
3. The constancy of flowers: Pollinators increase their reproductive success by reducing energy expenditure by visiting only blossoming plants [24],[25].

The global pollination is given as

(4)

are the t-th iteration solutions, is the Levy flight step, is a scaling factor, and the global best is the current optimal solution.

Local pollination is given by,

(5)

are randomly generated numbers and is a random number between 0 and 1. Different types of pollination

types and the Flowchart of FPA is as shown in Figures 1 and 2, respectively.

A diagram of a flower

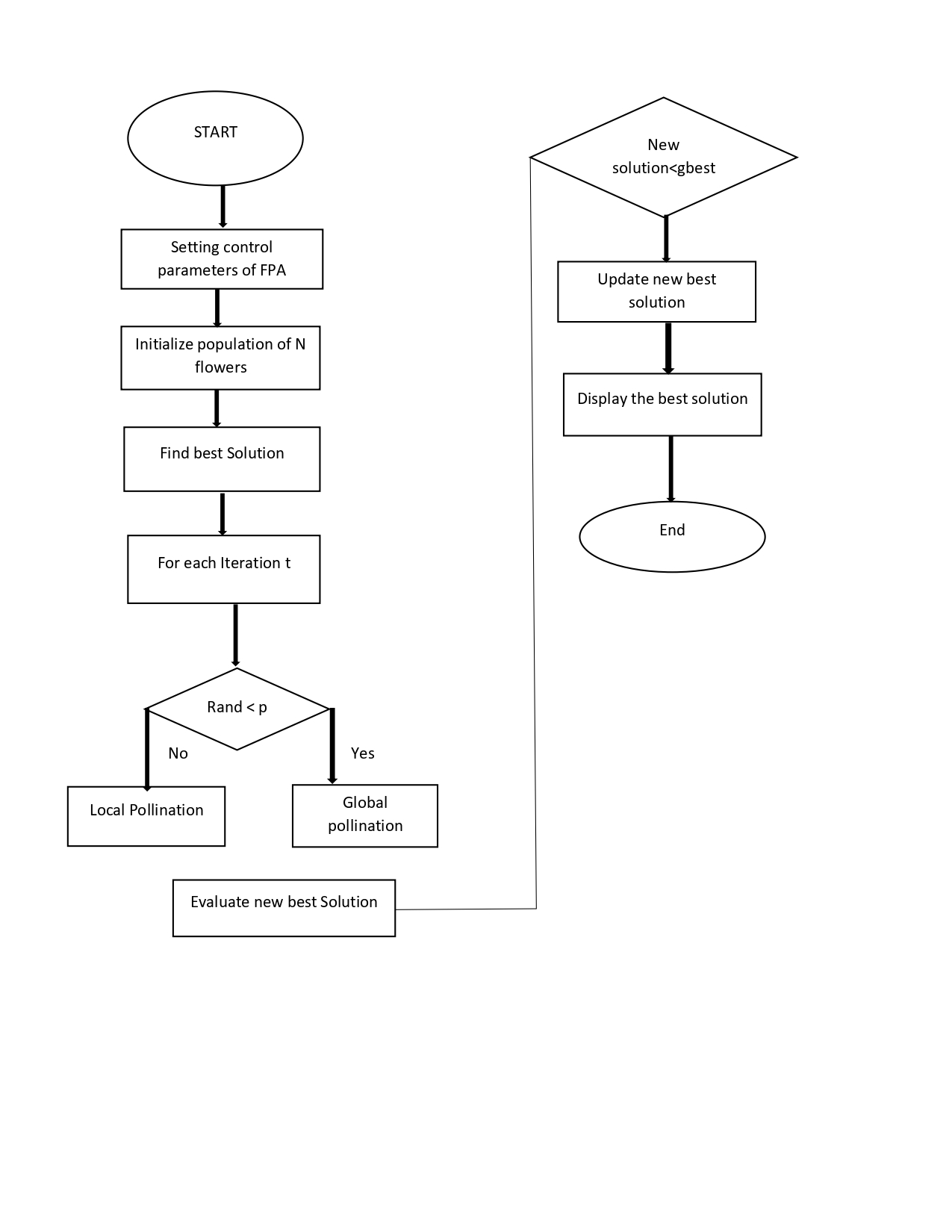
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**FIGURE 2.** The pollinators and pollination types

## BAT Algorithm (BAT)

Yang introduced the Bat algorithm in 2010 (see Figure 3). The fundamental Bat algorithm is inspired by biological processes. It is founded on the attributes of bat biosonar or echolocation. Wild bats transmit ultrasonic vibrations into the environment to facilitate hunting or navigation. The Bat algorithm has exhibited remarkable efficacy in addressing continuous optimization challenges [17],[18].

The bat algorithm (BAT) was initially introduced, which is associated with benchmark functions; thus, BAT executes particle swarm optimization and genetic algorithms. BA has been effectively applied to complex optimization challenges, including motor wheel optimization issues, clustering problems, and renowned engineering optimization tasks. BA demonstrates that the authors were compelled to use this framework for the assignment of attribute reduction within the specified literature. Bats are winged mammals possessing the ability of echolocation, often known as biosonar.



**FIGURE 3.** Flow chart of FPA

Echolocating creatures emit sounds to assess their environment and focus on the resulting echoes. The echoes will be utilized to identify and recognize the devices. Among all bat species, microbats extensively utilize echolocation. Echolocation in microbats is a form of sonar utilized to detect targets, avoid nearby obstacles in darkness, and identify roosting niches [20],[21],[22].

At a certain point in echolocation, microbats release a series of short, high-frequency sounds and then focus on the echoes that return from surrounding objects. A bat can ascertain an object’s dimensions, form, orientation, duration, and motion by echolocation. As bats approach their food, the rate of pulse emission can increase to as much as 200 pulses per second [24],[25]. A consistent persistence can be detected in each pulse. The wavelengths of a pulse have been commensurate with the sizes of their prey. The volume when searching for prey exceeds that when homing in on the prey. Conversely, the volume diminishes even as one approach the victims.

Based on the aforementioned characteristics of bat echolocation, Yang formulated the BAT algorithm (see Figure 4) , which has three idealized principles: Bats utilize echolocation to gauge distance and can discern variations among their prey in a remarkable manner; the intensity of sound can fluctuate in numerous ways. Ray tracing, although computationally intensive, serves as a highly beneficial feature for computational geometry and other applications.

## Methods for Digital Differentiator Design using Metaheuristic Algorithms

### *Error Criterion*

Linear systems that do not change over time are digital IIR filters. For the most part, the following difference equation characterizes the IIR filter:

(6)

is the input signal and is the output signal. This allows us to generalize the following expression for the filter transfer function:

(7)

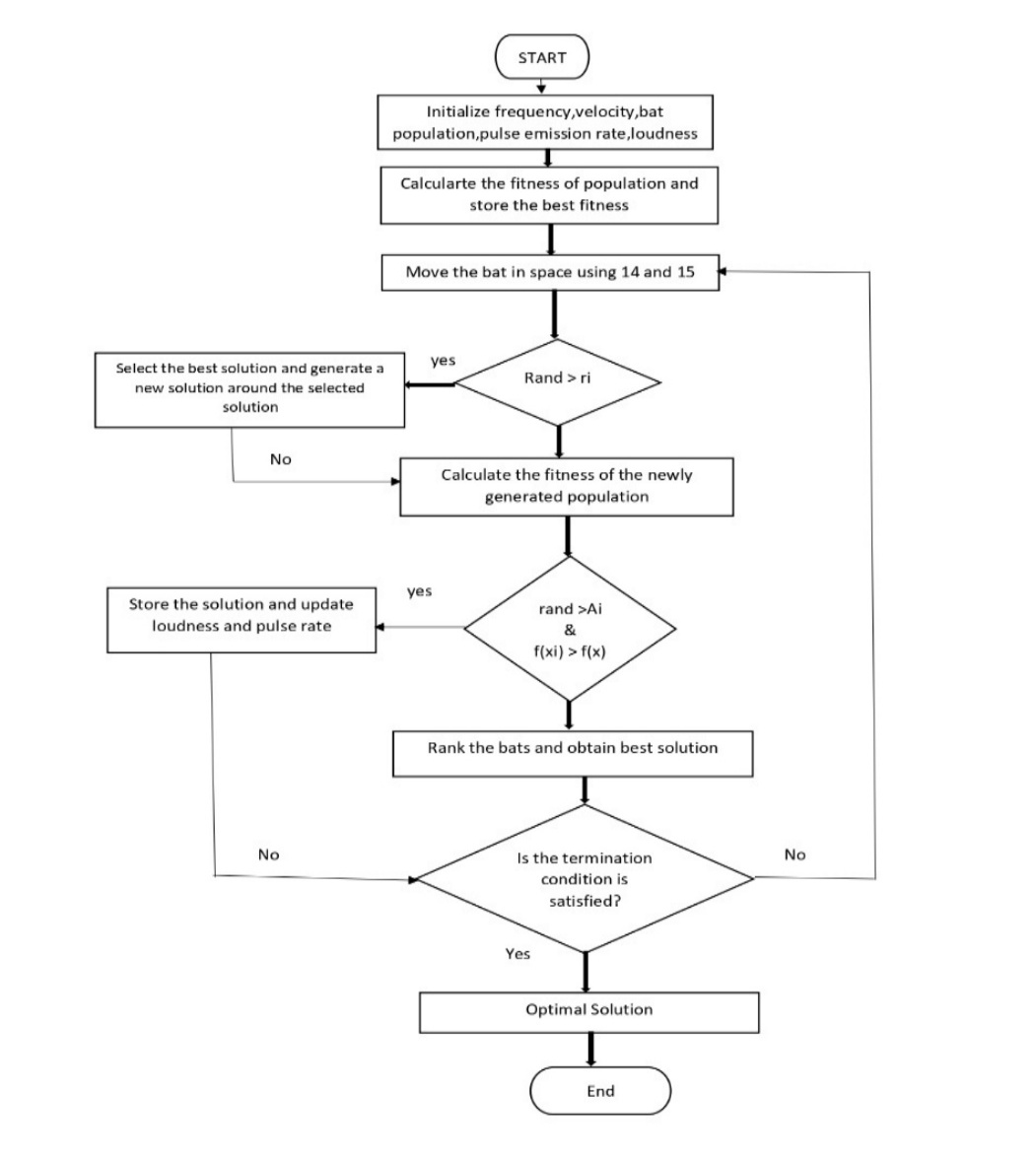
where the filter coefficients, denoted as and , define the filter’s properties. The frequency response of the IIR filter is described in the following way:

(8)

Consequently, the design of this filter is considered an optimization issue of the cost function defined as follows: minimum , defined as , represents the vector of filter coefficients (see Table 2). The objective is to reduce the cost function by adjusting . The cost function is typically expressed as follows

(9)

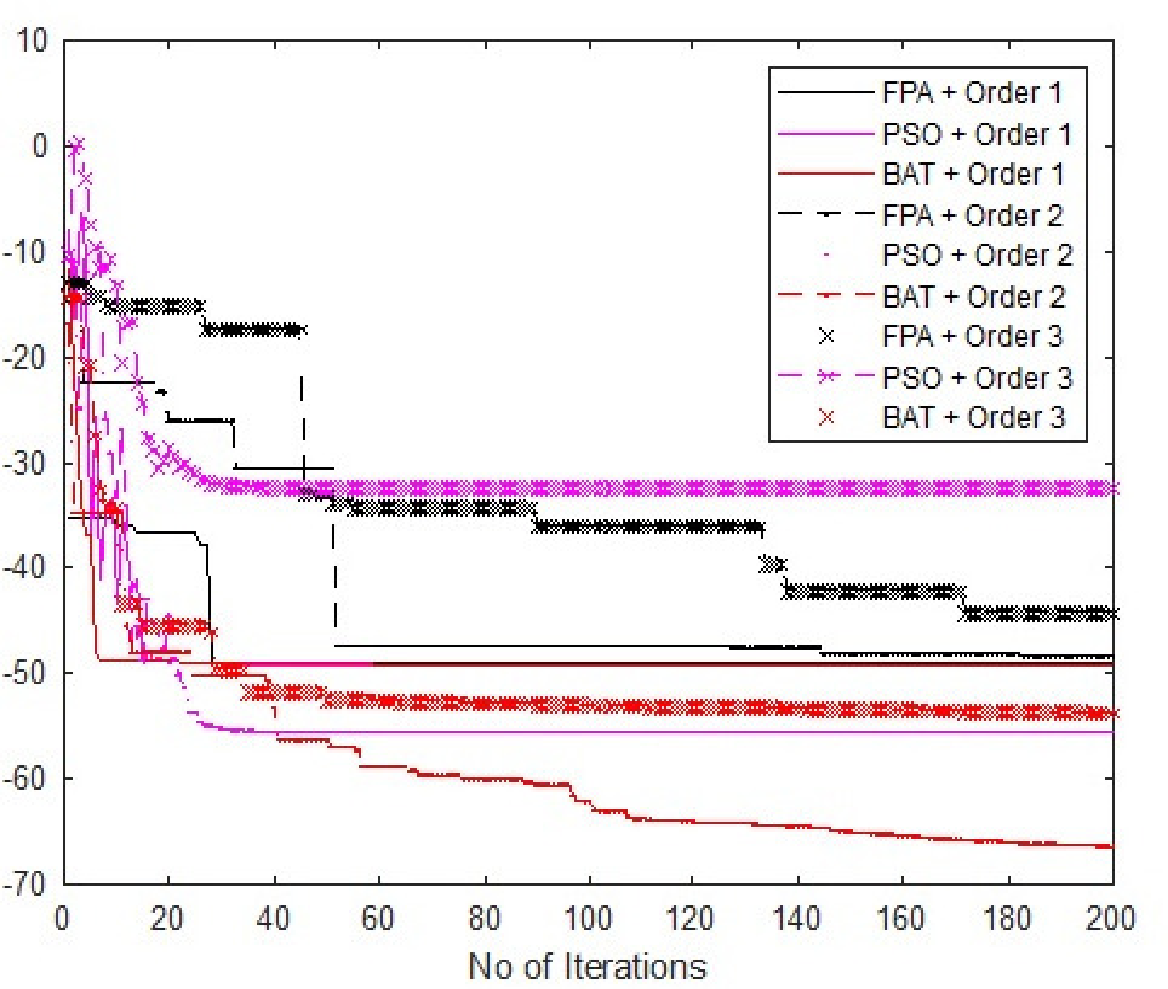
where represents the desired response and denotes the actual response of the filter, and signifies the number of samples utilized in the calculation of the cost function.



**FIGURE 4**. Flowchart of the BAT algorithm

**TABLE 2**. Third order filter coefficients using metaheuristic algorithms using error criterion

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algrithm | a0 | a1 | a2 | a3 | b0 | b1 | b2 | b3 |
| Error Criterion | -- | -- | -- | -- | -- | -- | -- | -- |
| BAT | -0.667246523306481 | -0.624795027524380 | -- | -- | 0.573718102718664 | 0.131658940776614 | -- | -- |
| - | 0.510910705767098 | 0.443697098848862 | -0.925177468243984 | -- | -0.809109131382650 | -0.583894790305074 | -0.0540507351577383 | -- |
| - | 0.0915080235991404 | 0.973129611384272 | -0.933912857465317 | -0.124677017982258 | 0.0483385720943908 | - 0.0281764666364431 | 0.891204891584000 | 0.410984533408664 |
| FPA | 0.999381042891010 | -0.933966544574105 | -- | -- | -0.858302786972030 | -0.192540104371670 | -- | -- |
| - | 1 | 0.225489167835771 | -1 | -- | -0.158220061531161 | -1 | -0.913817625295756 | -- |
| - | -0.0821090253576093 | -0.654526477981057 | 0.755309941755652 | 0.0357401423146518 | -0.00329763735561349 | 0.0923772723873869 | 0.693195104614112 | 0.156149203807528 |
| PSO | -0.767833964160578 | 0.723353505958720 | -- | -- | 0.151827372563198 | 0.662059936181212 | -- | -- |
| - | -0.999957169845188 | 0.897294643173026 | 0.303002955849324 | -- | 0.0468260015507263 | 0.961965805247379 | 0.382892875842286 | -- |
| - | -0.297031346293778 | 0.920687874221165 | -0.352291522545630 | -0.521796757013368 | 0.600726349134389 | 0.868250296730732 | 0.104994060474798 | 0.166292876183703 |
| L1 Error Criterion | -- | -- | -- | -- | -- | -- | -- | -- |
| BAT | -0.9155 | 0.9968 | -- | -- | 0.8634 | 0.1719 | -- | -- |
| - | 1 | 0.2534 | -0.2268 | -- | -0.1220 | 0.0808 | 0.0549 | -- |
| - | 1 | -0.7176 | 0.2230 | -0.6202 | -0.5174 | - 0.3999 | -0.9644 | -0.1955 |
| FPA | 0.9949 | -0.996 | -- | -- | -0.1671 | -0.8721 | -- | -- |
| - | 1 | 0.7553 | 0.7833 | -- | 0.4665 | 0.2189 | 0.1473 | -- |
| - | -0.8051 | 0.8051 | 0.9768 | -0.7329 | 0.9559 | 0.6764 | -0.3754 | -0.1180 |
| PSO | 0.8599 | -0.7962 | -- | -- | -0.7294 | -0.1459 | -- | -- |
| - | 1 | 0.2984 | -0.0717 | -- | 0.0925 | 0.2262 | 0.0955 | -- |
| - | -0.9984 | 0.4852 | 0.7799 | -0.1055 | 0.4425 | 0.9504 | 0.0262 | -0.2597 |

**FIGURE 5.** Convergence Plot using Error Criterion

### Optimal Filter Design

The Lp norm of an N -dimensional vector (signal) x is defined as

(10)

The following are the special cases,

• L1 Norm is defined as,

(11)

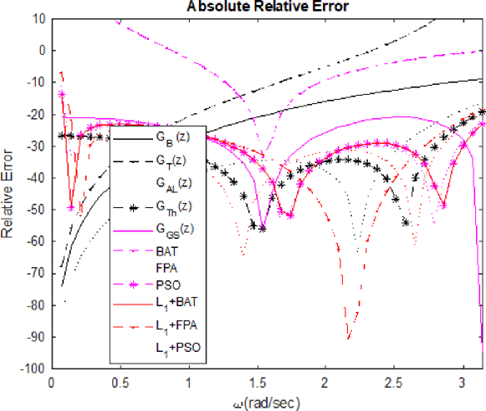
• L2 Norm is defined as,

(12)

A number of norms, including the Lp norms, are applicable to the distance method, an approximation problem (see Figure 6).

The computation of the gradient vector is limited to powers of even p. Finding the filter coefficients ak and bk is the primary goal of the digital IIR filter design procedure.An objective function that satisfies a number of fundamental characteristics and aims to minimize ψ(x) is defined as,

= (13)

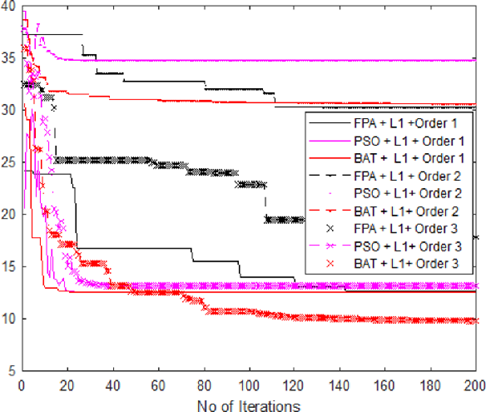


**FIGURE 6.** Convergence plot using *L*1 error criterion

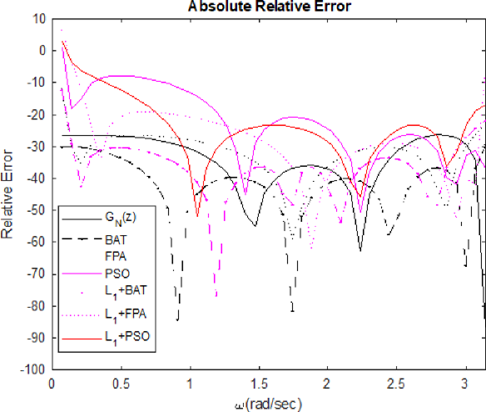
### Conclusion

This work investigated the design of IIR digital differentiators with three sophisticated metaheuristic optimization techniques: PSO, BAT, and FPA. The results showed that nature-inspired algorithms are a viable alternative to standard design methods, outperforming them in terms of frequency response accuracy and error minimization. FPA demonstrated the fastest convergence and lowest approximation error of the three techniques, making it an extremely successful strategy for digital differentiator design. The BAT algorithm likewise produced promising results, efficiently balancing exploration and exploitation, whereas PSO, despite its widespread use, performed significantly less accurately in comparison. The findings demonstrate the potential of bio-inspired optimization in improving digital signal processing jobs. Future research could focus on hybridizing these methods or incorporating machine learning approaches to improve the performance of digital differentiators in adaptive signal processing applications.

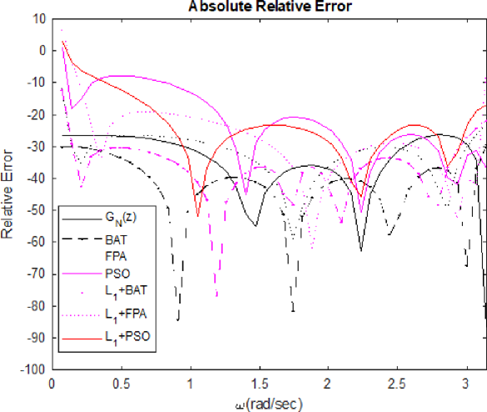
From Figures 7, 8, and 9, it indicates that the error obtained with the Flower Pollination Algorithm is less for second order and third order differentiators (-98dB, -83dB respectively). The error obtained for the first order differentiators with error criterion(-93dB) is less compared to other methods.



**FIGURE 7.** Error comparison of the proposed first order digital differentiator



**FIGURE 8.** Error comparison of the proposed second order digital differentiator

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**FIGURE 9.** Error comparison of the proposed third order digital differentiator

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